CHAPTER 5

Linear Functions

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Chapter Preview

Vocabulary

English/Spanish Vocabulary Audio Online:

English	Spanish
direct variation, *p. 301*	variación directa
linear equation, *p. 308*	ecuación lineal
piecewise function, *p. 348*	función de fragmentos
point-slope form, *p. 315*	forma punto-pendiente
rate of change, *p. 294*	tasa de cambio
slope, *p. 295*	pendiente
slope-intercept form, *p. 308*	forma pendiente-intercepto
standard form, *p. 322*	forma normal
step function, *p. 348*	función escalón
trend line, *p. 337*	línea de tendencia
x-intercept, *p. 322*	intercepto en *x*
y-intercept, *p. 308*	intercepto en *y*

BIG ideas

1 Proportionality

Essential Question: What does the slope of a line indicate about the line?

2 Functions

Essential Question: What information does the equation of a line give you?

3 Modeling

Essential Question: How can you make predictions based on a scatter plot?

DOMAINS

- Interpreting Functions
- Building Functions
- Interpreting Categorical and Quantitative Data

Get Ready!

Lesson 1-9 ◆ Solutions of a Two-Variable Equation

Tell whether the given ordered pair is a solution of the equation.

1. $4y + 2x = 3; (1.5, 0)$ **2.** $y = 7x - 5; (0, 5)$ **3.** $y = -2x + 5; (2, 1)$

Lesson 2-5 ◆ Transforming Equations

Solve each equation for y.

4. $2y - x = 4$ **5.** $3x = y + 2$ **6.** $-2y - 2x = 4$

Lesson 2-6 ◆ Comparing Unit Rates

7. Transportation A car traveled 360 km in 6 h. A train traveled 400 km in 8 h. A boat traveled 375 km in 5 h. Which had the fastest average speed?

8. Plants A birch tree grew 2.5 in. in 5 months. A bean plant grew 8 in. in 10 months. A rose bush grew 5 in. in 8 months. Which grew the fastest?

Lesson 4-4 ◆ Graphing a Function Rule

Make a table of values for each function rule. Then graph each function.

9. $f(x) = x + 3$ **10.** $f(x) = -2x$ **11.** $f(x) = x - 4$

Lesson 4-7 ◆ Arithmetic Sequences

Write an explicit formula for each arithmetic sequence.

12. $2, 5, 8, 11, \ldots$ **13.** $13, 10, 7, 4, \ldots$ **14.** $-3, -0.5, 2, 4.5, \ldots$

## Looking Ahead Vocabulary

15. A steep hill has a greater *slope* than a flat plain. What does the *slope* of a line on a graph describe?

16. Two streets are *parallel* when they go the same way and do not cross. What does it mean in math to call two lines *parallel*?

17. John was bringing a message to the principal's office when the principal *intercepted* him and took the message. When a graph passes through the y-axis, it has a *y-intercept*. What do you think a y-intercept of a graph represents?

Common Core Performance Task

Marbles in Water

Have you noticed that dropping objects into a glass of water raises the water level? The diagram and table below show what happens to the height of the water in a certain glass when different numbers of identical marbles are dropped into it.

15 cm

6.9 cm
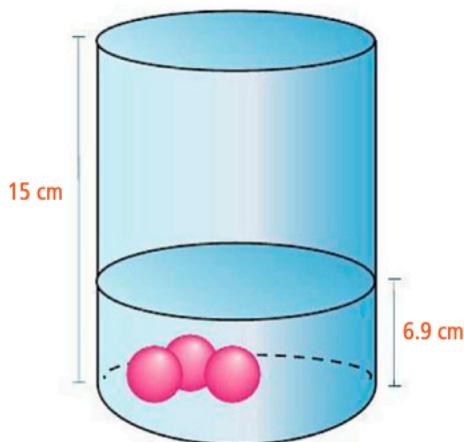
Number of Marbles, x	Height of Water (cm), y
3	6.9
5	7.5
9	8.7
14	10.2
17	11.1
23	12.9

Task Description

Predict the number of marbles you need to drop in the glass to raise the water level to the top of the glass.

Connecting the Task to the Math Practices

MATHEMATICAL PRACTICES

As you complete the task, you'll apply several Standards for Mathematical Practice.

- You'll analyze the data in the table to determine the type of relationship between the number of marbles in the glass and the water height. (MP 1)
- You'll model the data in the table with a function. (MP 2, MP 4)

Rate of Change and Slope

Getting Ready!

The table shows the horizontal and vertical distances from the base of the mountain at several poles along the path of a ski lift. The poles are connected by cable. Between which two poles is the cable's path the steepest? How do you know?

Drawing a diagram may help.

MATHEMATICAL PRACTICES

Pole	Horizontal Distance	Vertical Distance
A	20	30
B	40	35
C	60	60
D	100	70

handwritten annotations:
+20 +5
+20 +25
+40 +10

$\frac{+5}{+20} = \frac{1}{4}$

$\frac{+25}{+20} = \frac{5}{4}$ ✱
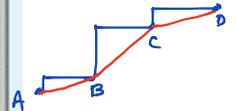
$\frac{+10}{+40} = \frac{1}{4}$

You can use ratios to show a relationship between changing quantities, such as vertical and horizontal change.

Rate of change shows the relationship between two changing quantities. When one quantity depends on the other, the following is true:

$$\text{rate of change} = \frac{\text{change in the } \textbf{\textit{dependent}} \text{ variable}}{\text{change in the } \textbf{\textit{independent}} \text{ variable}} = \frac{\Delta y}{\Delta x}$$

PROBLEM 1: FINDING RATE OF CHANGE USING A TABLE

a) The table shows the distance a band marches over time. Is the rate of change in distance with respect to time constant? What does the rate of change represent?

yes; The distance is increasing 260 ft/min.

Distance Marched

Time (min)	Distance (ft)	$\frac{\Delta \text{dist}}{\Delta \text{time}}$
1	260	$\frac{+260}{+1} = +260$
2	520	$\frac{+260}{+1} = +260$
3	780	$\frac{+260}{+1} = +260$
4	1040	$\frac{+260}{+1} = +260$

Constant rate of change

Determine whether each rate of change is constant. If it is find the rate of change and explain what it represents.

b) **Turtle Walking**

Time (min)	Distance (m)	$\frac{\Delta \text{dist}}{\Delta \text{time}}$
1	6	$\frac{+6}{+1} = +6$
2	12	$\frac{+3}{+1} = +3$
3	15	
4	21	

Not a constant rate of change

c) **Hot Dogs and Buns**

Hot Dogs	Buns	$\frac{\Delta \text{buns}}{\Delta \text{hot dogs}}$
1	1	$\frac{+1}{+1} = +1$
2	2	$\frac{+1}{+1} = +1$
3	3	$\frac{+1}{+1} = +1$
4	4	

constant

The buns are increasing 1 bun/hotdog.

d) **Airplane Descent**

| Time (min)	Elevation (ft)	$\frac{\Delta \text{elevation}}{\Delta \text{time}}$
0	30,000	$\frac{-1000}{+2} = -500$
2	29,000	$\frac{-1500}{+3} = -500$
5	27,500	$\frac{-3500}{+7} = -500$
12	24,000	

Constant

The elevation is decreasing 500 ft/min.

The graph of the ordered pairs (time, distance) in Problem 1a lie on a line, as shown at the right. The relationship between time and distance is linear. When data are linear, the rate of change is constant.

Notice also that the rate of change found in Problem 1a is the ratio of the vertical change (or *rise*) to the horizontal change (or *run*) between two points on the line. *The rate of change is called the __slope__ of the line.*

Distance Marched
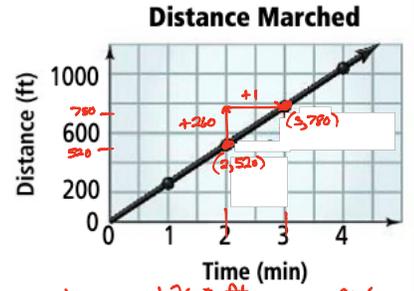
$$slope = m = \frac{vertical\ change}{horizontal\ change} = \frac{rise}{run}$$

$$\uparrow, \rightarrow : + \qquad \downarrow, \leftarrow : -$$

$$slope = \frac{+260\ ft}{+1\ min} = +260\ ft/min$$
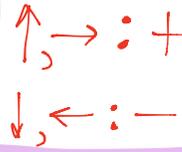
PROBLEM 2: FINDING SLOPE USING A GRAPH
Find the slope of each line.

a)

$$m = \frac{-4}{+2} = \boxed{-2} \qquad m = \frac{-2}{+1} = \boxed{-2}$$
$$m = \frac{+4}{-2} = \boxed{-2}$$

b)

$$m = \frac{+1}{+3} = \boxed{\frac{1}{3}}$$

c)

$$m = \frac{+4}{+1} = \boxed{4}$$

d)

$$m = \frac{+5}{+6} = \boxed{\frac{5}{6}}$$

e)

$$m = \frac{+6}{+8} = \boxed{\frac{3}{4}} \qquad m = \frac{-3}{-4} = \boxed{\frac{3}{4}}$$

f)
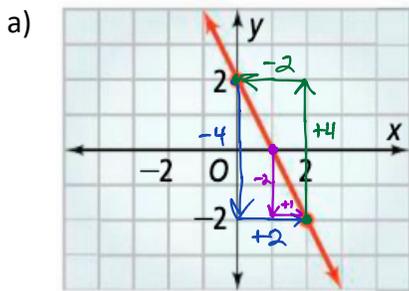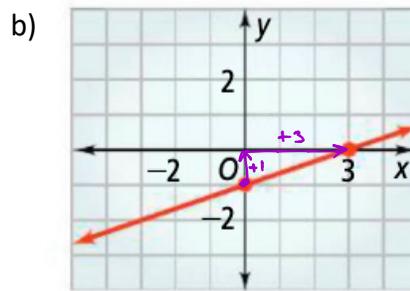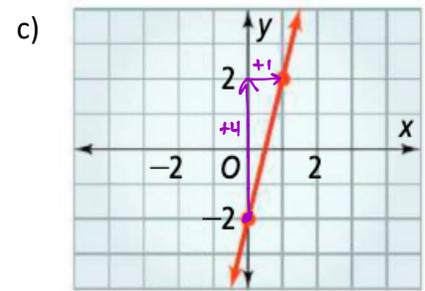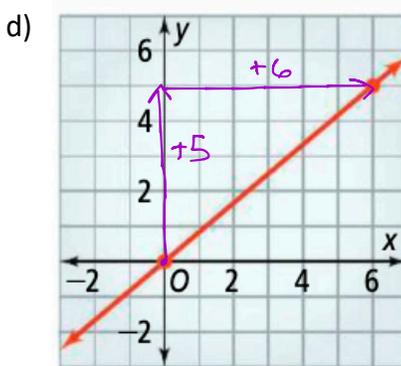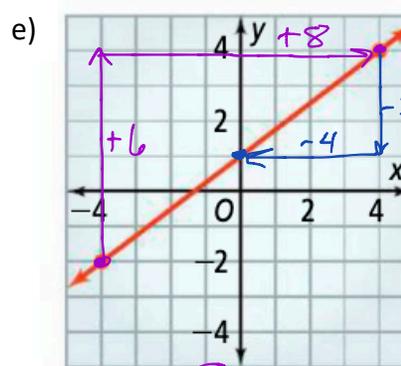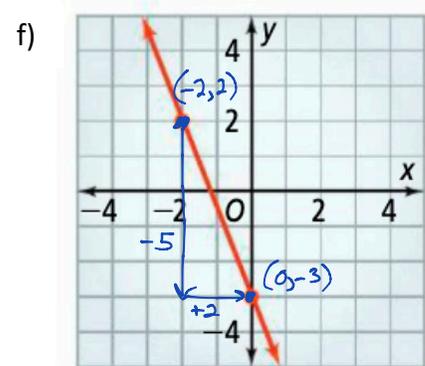
$$m = \frac{-5}{+2} = \boxed{-\frac{5}{2}}$$

$$m = \frac{Y_{new} - Y_{old}}{X_{new} - X_{old}}$$
$$= \frac{-3-2}{0+(-2)}$$
$$= \frac{-5}{+2}$$
$$= \boxed{-\frac{5}{2}}$$

You can use any two points on a line to find its slope. Use subscripts to distinguish between the two points. In the diagram, (x_1, y_1) are the coordinates of point *A*, and (x_2, y_2) are the coordinates of point *B*. To find the slope of $\overleftrightarrow{AB}$, you can use the *slope formula*.

KEY CONCEPT: THE SLOPE FORMULA
$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}, where\ x_2 - x_1 \neq 0$$

The x-coordinate you use first in the denominator must belong to the same ordered pair as the y-coordinate you use first in the numerator.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \; ; \; (x_1, y_1), (x_2, y_2)$$

PROBLEM 3: FINDING SLOPE USING POINTS

Find the slope of the line that passes through each pair of points.

a) $\overset{x_1 \; y_1}{(0,0)}, \overset{x_2 \; y_2}{(3,3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{3-0} = \frac{3}{3}$

$= \boxed{1}$

b) $\overset{x_1 \; y_1}{(4,4)}, \overset{x_2 \; y_2}{(5,3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-4}{5-4} = \frac{-1}{1}$

$= \boxed{-1}$

c) $\overset{x_1 \; y_1}{(-6,1)}, \overset{x_2 \; y_2}{(4,8)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-1}{4+6} = \boxed{\frac{7}{10}}$

d) $\overset{x_1 \; y_1}{(4.25,0)}, \overset{x_2 \; y_2}{(3.5,3)}$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{3.5-4.25} = \frac{3}{-0.75}$

$= \boxed{-4}$

e) $\overset{x_1 \; y_1}{(1,3)}, \overset{x_2 \; y_2}{(5,5)}$

$m = \frac{5-3}{5-1} = \frac{2}{4}$

$= \boxed{\frac{1}{2}}$

f) $\overset{x_1 \; y_1}{(0,-1)}, \overset{x_2 \; y_2}{(2,3)}$

$m = \frac{3+(+1)}{2-0} = \frac{4}{2}$

$= \boxed{2}$

g) $\overset{x_1 \; y_1}{(2,-3)}, \overset{x_2 \; y_2}{(5,-4)}$

$m = \frac{-4+(+3)}{5-2} = \frac{-1}{3}$

$= \boxed{-\frac{1}{3}}$

h) $\overset{x_1 \; y_1}{(-2,1)}, \overset{x_2 \; y_2}{(7,1)}$

$m = \frac{1-1}{7-(-2)} = \frac{0}{9}$

$= \boxed{0}$

PROBLEM 4: FINDING SLOPES OF HORIZONTAL AND VERTICAL LINES

Find the slope.

a)

$m = \frac{2-2}{2-(-3)} = \frac{0}{5}$

$= 0$
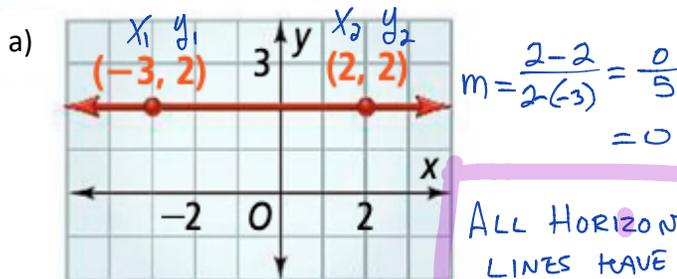
ALL HORIZONTAL LINES HAVE A SLOPE OF ZERO.

b)

$m = \frac{-2-1}{-2+(+2)} = \frac{-3}{0}$
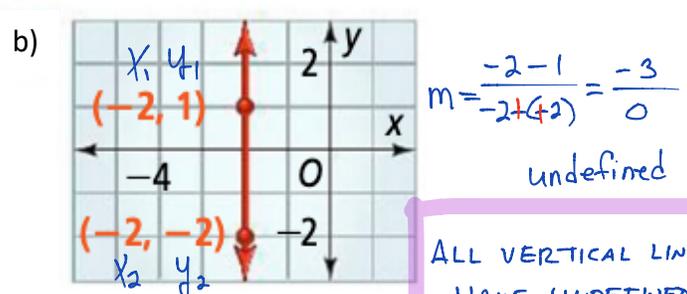
undefined

ALL VERTICAL LINES HAVE UNDEFINED SLOPES.

c) through the points $\overset{x_1 \; y_1}{(4,-3)}$ and $\overset{x_2 \; y_2}{(4,2)}$

$m = \frac{2-(-3)}{4-4} = \frac{5}{0}$

undefined

∴ vertical line

d) through the points $\overset{x_1 \; y_1}{(-1,-3)}$ and $\overset{x_2 \; y_2}{(5,-3)}$

$m = \frac{-3-(-3)}{5-(-1)} = \frac{0}{6}$

$= 0$

∴ horizontal line

## Concept Summary   Slopes of Lines

A line with positive slope slants upward from left to right.
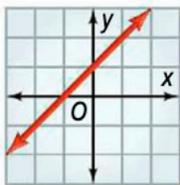
A line with negative slope slants downward from left to right.
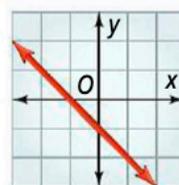
A line with a slope of 0 is horizontal.
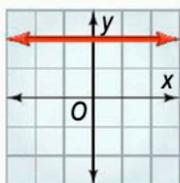
A line with an undefined slope is vertical.
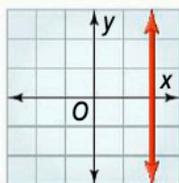
Lesson Check

Do you know HOW?

1. Is the rate of change in cost constant with respect to the number of pencils bought? Explain.

Cost of Pencils

Number of Pencils	1	4	7	12
Cost ($)	0.25	1	1.75	3

2. What is the slope of the line?
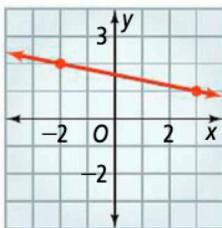
3. What is the slope of the line through $(-1, 2)$ and $(2, -3)$?

Do you UNDERSTAND?

MATHEMATICAL PRACTICES

4. Vocabulary What characteristic of a graph represents the rate of change? Explain.

5. Open-Ended Give an example of a real-world situation that you can model with a horizontal line. What is the rate of change for the situation? Explain.

6. Compare and Contrast How does finding a line's slope by counting units of vertical and horizontal change on a graph compare with finding it using the slope formula?

7. Error Analysis A student calculated the slope of the line at the right to be 2. Explain the mistake. What is the correct slope?

up 1 unit
right 2 units

Name _____   Period _____
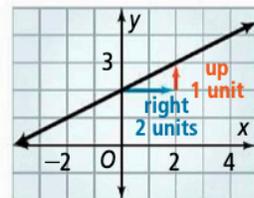
5-1 Practice Worksheet

Find the rate of change. Explain what the rate of change means for each situation.

1.

| People	Cost (dollars)
2	7.90
3	11.85
4	15.80
5	19.75
6	23.70
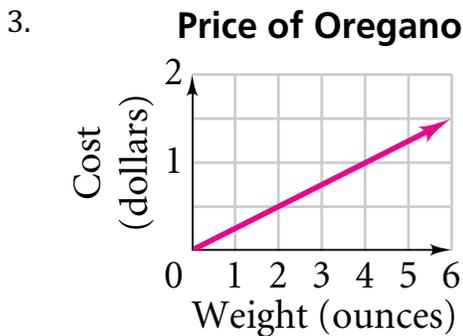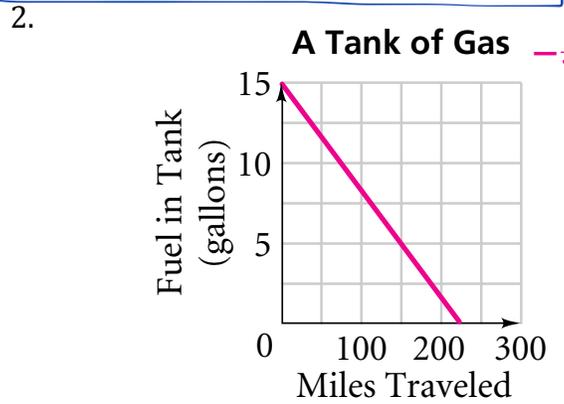
ces

nd Slope

2.

A Tank of Gas

3.

Price of Oregano

Find the slope of each line.

4.

5.
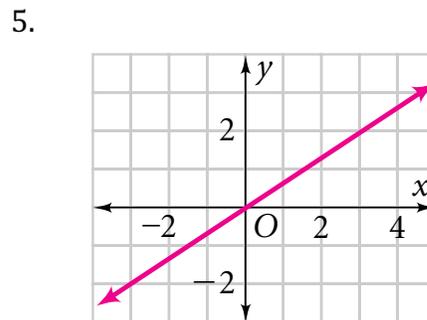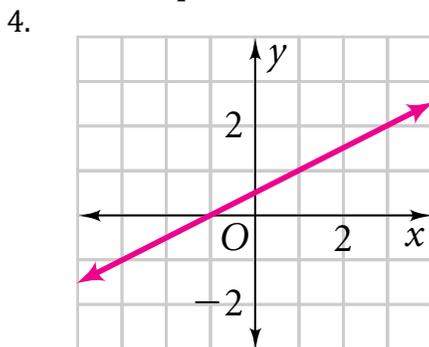
Find the slope of the line that passes through each pair of points.

6. (4,8), (8,11)

7. (-4, -5), (-9, 1)

8. (4, $1\frac{2}{3}$), (-2, $\frac{2}{3}$)

9. (-m, n), (3m, -n)

4

State whether the slope is zero or undefined.

10.
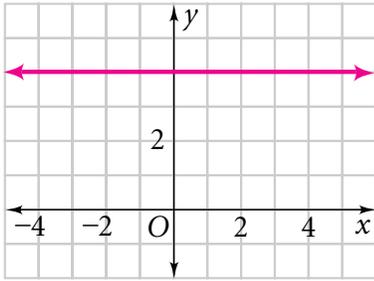
11. (4, 3), (4, -3)

(0, 18), (10, 27)

Find the rate of change in each situation.

12. A baby is 18 in. long at birth and 27 in. long at ten months.

13. The cost of group museum tickets is $48 for four people and $78 for ten people.

14. You drive 30 miles in one hour and 120 miles in four hours.

Through the given point, draw a line with the given slope.

15. (-2, 3); slope: $\frac{3}{5}$
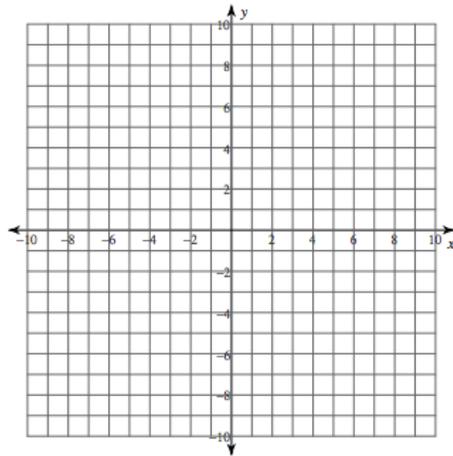
Find the slope of the sides of the figure.

16.
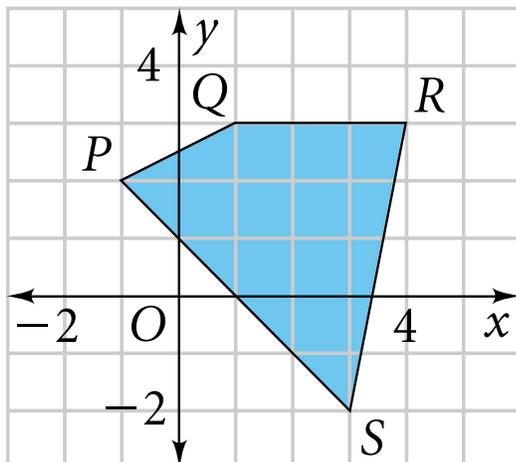
Find the value of *x* if the points lie on a line with the given slope.

17. $(x, 3), (2, 8)$; slope $= -\dfrac{5}{2}$

Tell whether each statement is *true* or *false*. If false, give a counterexample.

18. A rate of change must be either positive or zero.

19. All horizontal lines have the same slope.

20. A line with slope 1 always passes through the origin.

21. Two lines may have the same slope.

22. The slope of a line that passes through Quadrant III must be negative.

23. A line with slope 0 never passes through the point $(0, 0)$.

24. Two points with the same x-coordinate are always on the same vertical line.

Do the points in lie on the same line? How can you tell without graphing?

25. $D(-2, 3), E(0, 1), F(2, 1)$

Multiple Choice

26. A line has slope $\dfrac{4}{3}$. Through which two points could this line pass?

a) $(24, 19), (8, 10)$ b) $(10, 8), (16, 0)$ c) $(28, 10), (22, 2)$ d) $(4, 20), (0, 17)$

27. A horizontal line passes through the point $(5, 22)$. Which other point does the line contain?

a) $(5, 2)$ b) $(0, 22)$ c) $(22, 5)$ d) $(0, 5)$